Registration No.: | | | | | | | | | | |

Total Number of Pages: 02

Course: B.Tech/IDD

Sub_Code: 23BS1001

1st Semester Regular/Back Examination: 2024-25

SUBJECT: Mathematics-I

BRANCH(S): BIOMED, BIOTECH, CE, CE, CHEM, CIVIL, CSIT, AE, AEIE, AERO, AUTO, CSE, CSE, CSEAI, CSEAIML, CSEDS, CST, ECE, ECE, ETC, ELECTRONICS & C.E, EE, EEE, ELECTRICAL, ELECTRICAL & C.E, MANUTECH, IT, ME, MECH, METTA, MINERAL, MINING, CSEIOT, EEVDT, PLASTIC, MME

Time: 3 Hours

Max Marks: 100

Q.Code: R422

Answer Question No.1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III.

The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions: **(2 x 10)**

a) Find the length of the curve $y=\ln(\cos x); x=0 \; to \; x=\frac{\pi}{3}$.

b) Determine the value of $\Gamma\left(\frac{11}{2}\right)$.

c) In the Taylor's series expansion of e^x about $x=2$, find the coefficient of $(x-2)^4$.

d) A function $f(x)=1-x^2+x^3$ is defined in the closed interval [–1, 1]. Find the value of x in the open interval (–1, 1) for which the mean value theorem is satisfied.

e) Find the first order partial derivatives of $f(x,y)=\log(x^2+y^2)$.

f) Define sufficient condition for relative maxima for a function of two variables.

g) Check whether the set $S=\{(1,1,0),(0,1,0),(0,0,1)\}$ is LI or LD?

h) Are the vectors $\alpha_1=(1,0,-1)$, $\alpha_2=(1,2,1)$, $\alpha_3=(0,-3,2)$ forms a basis for R^3?

i) Let A and B be two square matrices of the same order and A is symmetric. Show that BAB^T is also symmetric.

j) State Caley-Hamilton theorem.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 x 8)**

a) Find the entire length of the curve $x=a(t+\sin t), y=a(1+\cos t)$.

b) Define improper integral. Test the convergency of $\int_{-\infty}^{\infty}\frac{dx}{x^2+a^2}$.

c) Find the volume of a sphere of radius r.

d) Discuss the applicability of Rolle's theorem to the function $f(x)=|x|$ in the interval (–1, 1).

e) Show that $\frac{x}{1+x} < \log(1+x) - \frac{x}{1+x}$ using mean value theorem.

f) Expand $\tan^{-1} x$ in powers of x by Maclaurin's theorem.

g) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, prove that $xu_x + yu_y = 0$.

h) Define maxima, minima, and saddle point. Find the maxima and minima of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$

i) Write the method of Lagrange's multipliers method. Find the extreme values of the function $f(x, y) = xy$ subject to $2x + 2y = 5$.

j) If $V = \{(x, y, z) : x = y\}$, then show that V is a vector space and find its basis and dimension.

k) Show that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix.

l) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$.

Part-III

Only Type Long Answer Questions (Answer Any Two out of Four)

Q3 **a)** Find the surface of the solid generated by the revolution of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis. **(8x2)**

b) Find the Maclaurin's series for $f(x) = e^x$. Also find the Maclaurin's series of $g(x) = \cosh x$.

Q4 **a)** If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. **(8x2)**

b) If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$.

Q5 **a)** Solve the system of linear equation by Gaussian elimination method: **(8x2)**

$$2x_1 + x_2 = 0, x_1 + 2x_2 + x_3 = 0, x_2 + 2x_3 + x_4 = 0, x_3 + 2x_4 = 5.$$

b) Let $U = \{(x_1, x_2, x_3, x_4): x_2 - 2x_3 + x_4 = 0\}$ and $W = \{(x_1, x_2, x_3, x_4): x_1 = x_4, x_2 = 2x_3\}$ be subspaces of $\mathbb{R}^4$. Then find basis and dimension of $U \cap W$.

Q6 **a)** Find inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 0 & 1 & 1 \end{bmatrix}$ using Gauss Jordan elimination. **(8x2)**

b) Define orthogonal matrix. Discuss about the eigenvalue of it.